

$$\text{In[1]:= } \mathbf{A} = \begin{pmatrix} -4 & 2 \\ 2 & -1 \end{pmatrix}$$

$$\text{Out[1]= } \{ \{-4, 2\}, \{2, -1\} \}$$

In[2]:= {v1[t], v2[t]} /. DSolve[
 $\mathbf{v1}'[t] == -4 \mathbf{v1}[t] + 2 \mathbf{v2}[t] + t^{-1}$ && $\mathbf{v2}'[t] == 2 \mathbf{v1}[t] - \mathbf{v2}[t] + 2 t^{-1} + 4,$
 $\{\mathbf{v1}[t], \mathbf{v2}[t]\}, t$
] // Expand // Flatten // MatrixForm

$$\text{Out[2]/MatrixForm=}$$

$$\begin{pmatrix} -\frac{8}{25} + \frac{8t}{5} + \frac{C[1]}{5} + \frac{4}{5} e^{-5t} C[1] + \frac{2C[2]}{5} - \frac{2}{5} e^{-5t} C[2] + \text{Log}[t] \\ \frac{4}{25} + \frac{16t}{5} + \frac{2C[1]}{5} - \frac{2}{5} e^{-5t} C[1] + \frac{4C[2]}{5} + \frac{1}{5} e^{-5t} C[2] + 2 \text{Log}[t] \end{pmatrix}$$

In[3]:= JordanDecomposition[A]

$$\text{Out[3]= } \{ \{-2, 1\}, \{1, 2\}, \{-5, 0\}, \{0, 0\} \}$$

$$\text{In[4]:= } \mathbf{DD} = \begin{pmatrix} 0 & 0 \\ 0 & -5 \end{pmatrix}; \mathbf{CC} = \begin{pmatrix} 1 & -2 \\ 2 & 1 \end{pmatrix}; (\mathbf{CCinv} = \text{Inverse}[\mathbf{CC}]) // \text{MatrixForm}$$

$$\text{Out[4]/MatrixForm=}$$

$$\begin{pmatrix} \frac{1}{5} & \frac{2}{5} \\ -\frac{2}{5} & \frac{1}{5} \end{pmatrix}$$

In[5]:= CCinv.{t⁻¹, 2 t⁻¹ + 4} // Simplify

$$\text{Out[5]= } \left\{ \frac{8}{5} + \frac{1}{t}, \frac{4}{5} \right\}$$

In[6]:= CC.{Log[t] + $\frac{8}{5}t + C_1$, $\frac{4}{25} + C_2 E^{-5t}$ } // Simplify // ColumnForm

$$\text{Out[6]= } -\frac{8}{25} + \frac{8t}{5} + \text{Log}[t] + C_1 - 2 e^{-5t} C_2$$

$$\frac{4}{25} + 2 \left(\frac{8t}{5} + \text{Log}[t] + C_1 \right) + e^{-5t} C_2$$

In[7]:= Ψ = $\begin{pmatrix} 1 & -2 E^{-5t} \\ 2 & E^{-5t} \end{pmatrix}$; Inverse[Ψ] // MatrixForm

$$\text{Out[7]/MatrixForm=}$$

$$\begin{pmatrix} \frac{1}{5} & \frac{2}{5} \\ -\frac{2 e^{5t}}{5} & \frac{e^{5t}}{5} \end{pmatrix}$$

In[8]:= Inverse[Ψ].{t⁻¹, 2 t⁻¹ + 4} // Simplify

$$\text{Out[8]= } \left\{ \frac{8}{5} + \frac{1}{t}, \frac{4 e^{5t}}{5} \right\}$$

In[9]:= Integrate[Inverse[Ψ].{t⁻¹, 2 t⁻¹ + 4}, t]

$$\text{Out[9]= } \left\{ \frac{8t}{5} + \text{Log}[t], \frac{4 e^{5t}}{25} \right\}$$

In[10]:= Ψ.Integrate[Inverse[Ψ].{t⁻¹, 2 t⁻¹ + 4}, t] // Expand

$$\text{Out[10]= } \left\{ -\frac{8}{25} + \frac{8t}{5} + \text{Log}[t], \frac{4}{25} + \frac{16t}{5} + 2 \text{Log}[t] \right\}$$